

# From de Sitter to de Sitter: A New Cosmic Scenario without Dark Energy

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## ABSTRACT

In the present lore of cosmology, matter and space-time emerged from a singularity and evolved through four different regimes: inflation, radiation, dark matter and dark energy dominated eras. In the radiation and dark matter dominated stages, the expansion of the Universe decelerates while the inflation and dark energy eras are accelerating regimes. So far there is no clear cut connection between these accelerating periods. More intriguing, the substance driving the present accelerating stage remains a mystery, and the best available candidate ( $\Lambda$ -vacuum) is plagued with the coincidence and cosmological constant problems.

In this paper we overcome such problems through an alternative cosmic scenario based on gravitationally-induced particle production. The model proposed here is non-singular with the space-time emerging from a pure initial de Sitter stage thereby providing a natural solution to the horizon problem. Subsequently, due to an instability provoked by the production of massless particles, the Universe evolves smoothly to the standard radiation dominated era thereby ending the production of radiation as required by the conformal invariance (Parker's theorem). Next, the radiation becomes subdominant with the Universe entering in the cold dark matter dominated era. Finally, the negative pressure associated with the creation of cold dark matter particles accelerates the expansion and drives the Universe to a final de Sitter stage. The late time cosmic expansion history is exactly like in the standard  $\Lambda$ CDM model, however, there is no dark energy. The model evolves between two limiting (early and late time) de Sitter regimes. Our scenario is fully determined by two

extreme energy densities, or equivalently, the associated de Sitter Hubble scales connected by  $\rho_I/\rho_f = (H_I/H_f)^2 \sim 10^{122}$ .

The microscopic description for gravitationally-induced particle production in an expanding Universe began with Schrödinger's [1] seminal paper, who referred to it as an alarming phenomenon. In the late 1960s, this issue was rediscussed by Parker and others [2, 3] based on the Bogoliubov mode-mixing technique in the context of quantum field theory in curved space-time. Physically, one may think that the (classical) time varying gravitational field works like a 'pump' supplying energy to the quantum fields.

In order to understand the basic approach, let us now consider a real minimally coupled massive scalar field  $\phi$  evolving in a flat expanding Friedman-Robertson-Walker (FRW) geometry. The field is described by the following action<sup>1</sup>

$$S = \frac{1}{2} \int \sqrt{-g} d^4x \left[ g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - m^2 \phi^2 \right]. \quad (1)$$

In terms of the conformal time  $\eta$  ( $dt = a(\eta)d\eta$ ), the metric tensor  $g_{\mu\nu}$  is conformally equivalent to the Minkowski metric  $\eta_{\mu\nu}$ , so that the line element is  $ds^2 = a^2(\eta)\eta_{\mu\nu}dx^\mu dx^\nu$ , where  $a(\eta)$  is the cosmological scale factor. Writing the field  $\phi(\eta, x) = a(\eta)^{-1}\chi$ , one obtains from the above action

$$\chi'' - \nabla^2 \chi + \left( m^2 a^2 - \frac{a''}{a} \right) \chi = 0, \quad (2)$$

where the prime denotes derivatives with respect to  $\eta$ . Notice that the field  $\chi$  obeys the same equation of motion as a massive scalar field in Minkowski space-time, but now with a time dependent *effective mass*,

$$m_{eff}^2(\eta) \equiv m^2 a^2 - \frac{a''}{a}. \quad (3)$$

This time varying mass accounts for the interaction between the scalar and the gravitational fields. The energy of the field  $\chi$  is not conserved (its action is explicitly time-dependent), and, more important, its quantization leads to particle creation at the expense of the classical gravitational background [2–4].

On the other hand, in the framework of general relativity theory (TRG), the scale factor of a FRW type Universe dominated by radiation satisfies the following equation:

$$a\ddot{a} + \dot{a}^2 = 0 \quad (4)$$

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<sup>1</sup> We adopt units such that  $\hbar = k_B = c = 1$ .

or, in the conformal time,  $a'' = 0$ . Therefore, for massless fields ( $m = 0$ ), there is no particle production since Eq. (2) reduces to the same of a massless field in Minkowski spacetime, and, as such, its quantization becomes trivial. This is the basis of Parker theorem concerning the absence of massless particle production in the early stages of the Universe. Note that Parker's result does not forbid the production of massless particles in a very early de Sitter stage ( $a'' \neq 0$ ). Potentially, we also see that massive particles can always be produced by a time varying gravitational field. As we shall see, such features are incorporated in the scenario proposed here.

In principle, for applications in cosmology, the above semiclassical results has three basic difficulties, namely:

- (i) The scalar field was treated as a test field, and, therefore, the FRW background is not modified by the newly produced particles.
- (ii) The particle production is an irreversible process, and, as such, it should be constrained by the second law of thermodynamics.
- (iii) There is no a clear prescription of how an irreversible mechanism of quantum origin can be incorporated in the Einstein Field Equations (EFE).

Later on, a possible macroscopic solution for these problems was put forward by Prigogine and coworkers [5] using non-equilibrium thermodynamics for open systems, and by Calvão, Lima & Waga [6] through a covariant relativistic treatment for imperfect fluids (see also [7]). The leitmotiv of the approach is that particle production, at the expense of the gravitational field, is an irreversible process constrained by the usual requirements of non-equilibrium thermodynamics. This irreversible process is described by a negative pressure term in the stress tensor whose form is constrained by the second law of thermodynamics<sup>2</sup>.

In comparison to the standard equilibrium equations, the irreversible creation process is described by two new ingredients: a balance equation for the particle number density and a negative pressure term in the stress tensor. Such quantities are related to each other in a very definite way by the second law of thermodynamics. Since the middle of the nineties, several interesting features of cosmologies with creation of cold dark matter and radiation have been investigated by many authors [8–11].

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<sup>2</sup> The semiclassical approach is unable to provide the entropy burst accompanying the particle production since it is adiabatic and reversible.

In this context, we are proposing here a new cosmological scenario where the accelerating stages of the cosmic evolution are powered uniquely by the creation of massless and massive cold dark matter particles. In this model, the Universe starts from a de Sitter dominated phase ( $a \propto e^{H_I t}$ ) powered by the production of massless particles. Subsequently, it deflates and evolves to the standard radiation phase ( $a \propto t^{1/2}$ ) thereby ending the creation of massless particles. Due to expansion, the radiation becomes subdominant with the Universe entering in the cold dark matter (CDM) dominated era. Finally, the negative pressure associated with the creation of cold dark matter particles accelerates the expansion and drives the Universe to a final de Sitter stage. The horizon problem is naturally solved in the initial de Sitter phase. In addition, the transition from Einstein-de Sitter ( $a \propto t^{2/3}$ ) to a de Sitter final stage ( $a \propto e^{H_f t}$ ) guarantee the consistence of the model with the supernovae type Ia data and complementary observations. A transition redshift of the order of a few (exactly the same value predicted by  $\Lambda$ CDM) is also obtained.

For simplicity, let us consider the EFE for a flat geometry:

$$8\pi G\rho = 3\frac{\dot{a}^2}{a^2}, \quad 8\pi G(p + p_c) = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}, \quad (5)$$

where an overdot means time derivative,  $\rho$  and  $p$  are the dominant energy density and pressure of the cosmic fluid, respectively, and  $p_c$  is a dynamic pressure which depends on the particle production rate. Special attention has been paid to the simpler process termed “adiabatic” particle production. It means that particles and entropy are produced in the space-time, but the specific entropy (per particle),  $\sigma = S/N$ , remains constant [6]. In this case, the creation pressure reads [6-9]

$$p_c = -\frac{(\rho + p)\Gamma}{3H}, \quad (6)$$

where  $\Gamma$  with dimensions of  $(time)^{-1}$  is the particle production rate and  $H = \dot{a}/a$  is the Hubble parameter.

*How the evolution of  $a(t)$  is affected by  $\Gamma$ ?* By assuming a dominant cosmic fluid satisfying the equation of state (EoS),  $p = \omega\rho$ , where  $\omega$  is a constant, the EFE imply that

$$\dot{H} + \frac{3}{2}(1 + \omega)H^2 \left(1 - \frac{\Gamma}{3H}\right) = 0. \quad (7)$$

The de Sitter solution ( $\dot{H} = 0$ ,  $\Gamma = 3H = constant$ ) is now possible regardless of the EoS defining the cosmic fluid. Since the Universe is evolving, such a solution is unstable, and, as long as  $\Gamma \ll 3H$ , conventional solutions without particle production are recovered.

$\Rightarrow$  The main effect of  $\Gamma$  is to provoke a dynamic instability in the space-time, thereby allowing a transition from de Sitter to a conventional solution, and vice versa.

### A. From an early de Sitter stage to the standard radiation phase

Let us first discuss the transition from an initial de Sitter stage to the standard radiation phase. The main theoretical constraints are:

- *The model must not only solve the horizon problem but also provide a quasiclassical boundary condition to quantum cosmology (a hint on how to solve the initial singularity problem).*
- *Massless particles cannot be quantum-mechanically produced in the conventional radiation phase (Parker's Theorem).*

To begin with, let us assume a radiation dominated Universe ( $\omega = 1/3$ ,  $\Gamma \equiv \Gamma_r$ ). The dynamics is determined by the ratio  $\Gamma_r/3H$  (see (7)). The most natural choice would be a ratio which favors no epoch in the evolution of the Universe ( $\Gamma_r/3H = \text{constant}$ ). However, the particle production must be strongly suppressed,  $\Gamma_r/3H \ll 1$ , when the Universe enters the radiation phase. This means that the expansion The simplest formula satisfying such a criterion is linear, namely:  $\Gamma_r/3H = H/H_I$ , where  $H_I$  is the initial de Sitter expansion rate ( $H \leq H_I$ ). Inserting this into (7) it becomes:

$$\dot{H} + 2H^2 \left(1 - \frac{H}{H_I}\right) = 0. \quad (8)$$

The solution of the above equation can be written as

$$H(a) = \frac{H_I}{1 + Da^2}, \quad (9)$$

where  $D \geq 0$  is an integration constant. Note that  $H = H_I$  is a special solution of Eq.(8) describing the exponentially expanding de Sitter space-time. This solution is unstable with respect to the critical value  $D = 0$ . For  $D > 0$ , the universe starts without a singularity and evolves continuously towards a radiation stage,  $a \sim t^{1/2}$ , when  $Da^2 \gg 1$ . By integrating (9), we obtain the scale factor:

$$H_I t = \ln \frac{a}{a_*} + \frac{\lambda^2}{2} (a/a_*)^2, \quad (10)$$

where  $\lambda^2 = Da_*^2$  is an integration constant and  $a_*$  defines the transition from the de Sitter stage to the beginning of the standard radiation epoch<sup>3</sup>. At early times ( $a \ll a_*$ ), when the logarithmic term dominates, one finds  $a \simeq a_* e^{H_I t}$ , while at late times,  $a \gg a_*$ ,  $H \ll H_I$ , (10) reduces to  $a \simeq a_* \left(\frac{2H_I t}{\lambda^2}\right)^{1/2}$ , and the standard radiation phase is reached.

It should be noticed that the time scale  $H_I^{-1}$  provides the greatest value of the energy density,  $\rho_I = \frac{3H_I^2}{8\pi G}$ , characterizing the initial de Sitter stage which is supported by the maximal radiation production rate,  $\Gamma_r = 3H_I$ . From (5) and (9) we obtain the radiation energy density:

$$\rho_r = \rho_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-2}. \quad (11)$$

As expected, we see again that the conventional radiation phase,  $\rho_r \sim a^{-4}$ , is attained when  $a \gg a_*$ .

*How the cosmic temperature evolves?* For “adiabatic” particle production the energy density scales as  $\rho_r \sim T^4$  [6, 7], and the above equation implies that

$$T_r = T_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-1/2}, \quad (12)$$

where  $T_I$  is the temperature of the initial de Sitter phase which must be uniquely determined by the scale  $H_I$ . We see that the expansion proceeds isothermally during the de Sitter phase ( $a \ll a_*$ ). A basic consequences is:

$\Rightarrow$  *The supercooling and subsequent reheating taking place in several inflationary variants are avoided [15, 16]. In other words, there is no the so-called ‘graceful exit’ problem.*

After de Sitter stage, the temperature decreases continuously in the course of the expansion. For  $a \gg a_*$  ( $H \ll H_I$ ), we obtain  $T \sim a^{-1}$ . Accordingly, the comoving number of photons becomes constant since  $n \propto a^{-3}$ , as expected for the standard radiation stage.<sup>4</sup>

*What about the initial temperature  $T_I$ ?* Since the model starts as a de Sitter space-time, the most natural choice is to define  $T_I$  as the Gibbons-Hawking temperature [18] of

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<sup>3</sup> This kind of evolution was first discussed by G. L. Murphy [12] by studying possible effects of the second viscosity in the very early Universe. Later on, it was also investigated in a more general framework involving cosmic strings by J. D. Barrow [13] who coined the expression “Deflationary Universes”. It has also been discussed in connection with decaying  $\Lambda(t)$ -models [14].

<sup>4</sup> Since  $n_r \propto T^3$ , the average photon concentration reads  $n_r = n_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-\frac{3}{2}}$ .

its event horizon,  $T_I = H_I/2\pi$ . Naively, one may expect  $T_I$  of the same order or smaller than the Planck temperature because of the classical description. From EFE we have  $\rho_I = 3m_{Pl}^2 H_I^2/8\pi$  (where  $m_{Pl} \simeq 1.22 \times 10^{19} GeV$ ), and since the energy density is  $\rho_I = N_*(T)T_I^4$ , one finds  $T_I \sim H_I \sim 10^{19} GeV$  (where  $N_*(T) = \pi^2 g_*(T)/30$  depends on the number of effectively massless particles).

Naturally, due to the initial de Sitter phase, the model is free of particle horizons. A light pulse beginning at  $t = -\infty$  will have traveled by the cosmic time  $t$  a physical distance,  $d_H(t) = a(t) \int_{-\infty}^t \frac{d\tilde{t}}{a(\tilde{t})}$ , which diverges thereby implying the absence of particle horizons:

$\Rightarrow$  *The local interactions may homogenize the whole Universe.*

Since photons are not produced in the radiation phase, the Big-Bang Nucleosynthesis (BBN) may work in the conventional way [17]. Subsequently, the Universe enters the cold dark matter (Einstein-de Sitter,  $a(t) \propto t^{2/3}$ ) dominated phase.

## B. From Einstein-de Sitter to a late time de Sitter stage

Due to conservation of baryon number the remaining question is the production rate of cold dark matter particles and the overall late time evolution. In other words, what is the form of  $\Gamma_{dm}$ ? For simplicity, we consider here only the dominant CDM component.

In principle,  $\Gamma_{dm}$  should be determined from quantum field theory in curved spacetimes. In the absence of a rigorous treatment, we consider (phenomenologically) the following fact [19–21]:

- *All available observations are in accordance with the  $\Lambda$ CDM evolution both at the background and perturbative levels.*

Now, we recall that a flat  $\Lambda$ CDM model evolves like:

$$\dot{H} + \frac{3}{2}H^2 \left[ 1 - \left( \frac{H_f}{H} \right)^2 \right] = 0, \quad (13)$$

where  $H_f^2 = \Lambda/3$  sets the Hubble scale of the final de Sitter stage ( $H \geq H_f$ ). Such behavior should be compared to that predicted for a dust filled model ( $\omega = 0$ ,  $\Gamma \equiv \Gamma_{dm}$ ) with particle production (see Eq. (7)):

$$\dot{H} + \frac{3}{2}H^2 \left( 1 - \frac{\Gamma_{dm}}{3H} \right) = 0. \quad (14)$$

By comparing (13) and (14), we see that the same background evolution requires that  $\Gamma_{dm}/3H = (H_f/H)^2$ . The limiting value of the creation rate,  $\Gamma_{dm} = 3H_f$ , leads to a late



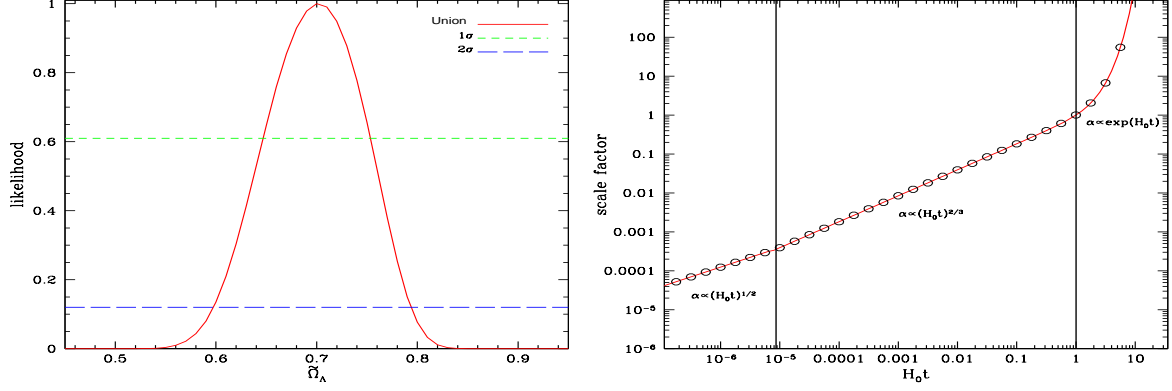


FIG. 1: **(a)** The likelihood for  $\tilde{\Omega}_\Lambda$  based on 307 Supernova data (Union)[19]. **(b)** Evolution of the scale factor predicted by the matter creation model (solid line) and the traditional  $\Lambda$ CDM cosmology (open points). In this plot we have adopted the best fit,  $\tilde{\Omega}_\Lambda = 0.72$ , from Supernova data.

time de Sitter phase ( $\dot{H} = 0$ ,  $H = H_f$ ) thereby showing that the de Sitter solution now becomes an attractor at late times. With this proviso, the solution of (14) reads:

$$H^2 = H_0^2 \left[ (1 - \tilde{\Omega}_\Lambda)(1 + z)^3 + \tilde{\Omega}_\Lambda \right], \quad (15)$$

where  $\tilde{\Omega}_\Lambda \equiv (H_f/H_0)^2$  is smaller than unity and  $1 + z = a^{-1}$ . Such solution mimics the Hubble function  $H(z)$  of the traditional flat  $\Lambda$ -cosmology, with  $\tilde{\Omega}_\Lambda$  playing the dynamical role of  $\Omega_\Lambda$  (dark energy appearing in the concordance model). The dark matter parameter ( $\Omega_{dm} = 1$ ) is also replaced by an effective parameter,  $(\Omega_{dm})_{eff} \equiv 1 - \tilde{\Omega}_\Lambda$ , which quantifies the amount of matter that is clustering. This explains why this model is in agreement with the dynamical determinations related to the amount of the cold dark matter at the cluster scale, and, simultaneously, may also be compatible with the position of the first acoustic peak in the pattern of CMB anisotropies which requires  $\Omega_{total} = 1$ .

By integrating (15) we obtain:

$$a(t) = \left( \frac{1 - \tilde{\Omega}_\Lambda}{\tilde{\Omega}_\Lambda} \right)^{1/3} \sinh^{2/3} \left( \frac{3H_0\sqrt{\tilde{\Omega}_\Lambda}}{2} t \right). \quad (16)$$

$\Rightarrow$  The late time dynamics is determined by a single parameter ( $\tilde{\Omega}_\Lambda$ ) and is identical to that predicted by the flat  $\Lambda$ CDM model.

In Figure 1, we show the likelihood of  $\tilde{\Omega}_\Lambda$  based on the Union supernova sample. Note also that by replacing the value of  $\Gamma_{dm}$  into the definition of the creation pressure (see Eq. 6) one obtains that it is negative and constant ( $p_c = -3H_f^2/8\pi G = -3\tilde{\Omega}_\Lambda H_0^2/8\pi G$ ). Therefore, the late time evolution of our complete cosmological scenario coincides exactly with the one recently discussed in Refs. [22, 23] following a slightly different approach.

Concluding, a new cosmology based on the production of massless particles (in the early de Sitter phase) and CDM particles (in the transition to a late time de Sitter stage) has been discussed. The same mechanism avoids the initial singularity, particle horizon and the late time coincidence problem of the  $\Lambda$ CDM model has been eliminated ( $\Lambda \equiv 0$ ).

In this scenario, the standard cosmic phases - a radiation era followed by an Einstein-de Sitter evolution driven by nonrelativistic matter until redshifts of the order of a few - are not modified. However, the model has two extreme accelerating phases (very early and late time de Sitter phases) powered by the same mechanism (particle creation). Therefore, it sheds some light on a possible connection among the different accelerating stages of the universe. In particular, since  $H_f^2 = \tilde{\Omega}_\Lambda H_0^2$ , where  $\tilde{\Omega}_\Lambda \sim 0.7$  and  $H_0 \simeq 1.5 \times 10^{-42} \text{ GeV}$ , it sets the ratio of the primeval and late time de Sitter scales to be  $\rho_I/\rho_f = (H_I/H_f)^2 \sim 10^{122}$ . Such a result in the present context has no correlation with the so-called cosmological constant problem [14, 25].

As it appears, the cosmic history discussed here is semi-classically complete. However, there is no guarantee that the initial de Sitter configuration is not only the boundary condition of a true quantum gravitational effect. In other words, the very early de Sitter phase may be the result of a quantum fluctuation which is further semi-classically supported by the creation of massless particles (in this connection see [24] and Refs. there in).

Naturally, the existence of an early isothermal de Sitter phase suggests that thermal fluctuations (within the de Sitter event horizon) may be the causal origin of the primeval seeds that will form the galaxies. Such a possibility and its consequences for the structure formation problem deserves a closer investigation and is clearly out of the scope of the present paper.

At present, we also know that a more complete version of the late time evolution must be filled with CDM ( $\sim 96\%$ ) and baryons ( $\sim 4\%$ ), and, unlike  $\Lambda$ -cosmology, the baryon to dark matter ratio is a redshift function [22, 23]. In particular, this means that studies involving the gas mass fraction may provide a crucial test of our scenario, potentially, modifying our

present view of the dark sector. Some investigations along the above discussed lines are in progress and will be published elsewhere.

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- [1] E. Schrödinger, *Physica* **6**, 899 (1939).
  - [2] L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968); *ibidem*, *Phys. Rev.* **183**, 1057 (1969).
  - [3] N. D. Birrell & P. C. Davies, *Quantum Fields in Curved Space*, CUP, Cambridge (1982); A. A. Grib, S. G. Mamayev and V. M. Mostepanenko, *Vacuum Quantum Effects in Strong Fields*, Friedmann Laboratory Publishing, St. Petersburg (1994).
  - [4] V. F. Mukhanov and S. Winitzki, *Introduction to Quantum Effects in Gravity*, Cambridge University Press, Cambridge, (2007).
  - [5] I. Prigogine *et al.*, *Gen. Rel. Grav.* **21**, 767 (1989).
  - [6] M. O. Calvão, J. A. S. Lima, I. Waga, *Phys. Lett. A* **162**, 223 (1992). See also, J. A. S. Lima, M. O. Calvão, I. Waga, “Cosmology, Thermodynamics and Matter Creation”, *Frontier Physics, Essays in Honor of Jayme Tiomno*, World Scientific, Singapore (1990), [arXiv:0708.3397].
  - [7] J. A. S. Lima, A. S. M. Germano, *Phys. Lett. A* **170**, 373 (1992).
  - [8] W. Zimdahl and D. Pavón, *Phys. Lett. A* **176**, 57 (1993); W. Zimdahl and D. Pavón, *Mon. Not. R. Astr. Soc.* **266**, 872 (1994); W. Zimdahl and D. Pavón, *GRG* **26**, 1259 (1994); J. Gariel and G. Le Denmat, *Phys. Lett. A* **200** 11 (1995);
  - [9] R. A. Susmann, *Class. Q. Grav.* **11**, 1445 (1994); J. Gariel, M. Cissoko, *Class. Quant. Grav.* **13**, 2977 (1996); L. R. W. Abramo and J. A. S. Lima, *Class. Quantum Grav.* **13**, 2953 (1996), gr-qc/9606064; J. A. S. Lima, A. S. M. Germano and L. R. W. Abramo, *Phys. Rev. D* **53**, 2953 (1996), gr-qc/9511006,
  - [10] V. B. Johri and S. K. Pandey, *Int. J. Theor. Phys.* **38**, 1981 (1999); T. Harko and M. K.

- Mak, *Astrophys. Space Sci.* **253**, 161 (1997); J. A. S. Lima, *Gen. Rel. Grav.* **29**, 805 (1997); K. Desinkhan, *Gen. Rel. Grav.* **29**, 435 (1997); *Class. Quant. Grav.* **16**, 2741 (1999); J. A. S. Lima and L. R. W. Abramo, *Phys. Lett. A*, **257**, 123 (1999); M. K. Mak and T. Harko, *Class. Quant. Grav.* **16**, 4085 (1999); *ibidem*, *IJMP D* **8** 607 (1999); J. A. S. Lima and U. F. Wichoski, *Phys. Lett. A* **262**, 103 (1999); W. Zimdahl, *Phys. Rev. D* **61** 083511 (2000); J. A. S. Lima, A. I. Silva and S. M. Viegas, *Mon. Not. R. Astron. Soc.* **312**, 747 (2000), [astro-ph/9902337](#); W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavón, *Phys. Rev. D* **64**, 063501 (2001).
- [11] J. A. S. Lima and J. S. Alcaniz, *Astron. Astrophys.* **348**, 1 (1999); J. A. S. Lima, F. E. Silva, R. C. Santos, *Class. Quant. Grav.* **25**, 205006 (2008), [arXiv:0807.3379 \[astro-ph\]](#); G. Steigman, R. C. Santos and J. A. S. Lima, *JCAP* **06**, 033 (2009), [[arXiv:0812.3912](#)].
- [12] G. L. Murphy, *Phys. Rev.* **8**, 4231 (1973).
- [13] J. D. Barrow, *Phys. Lett. B* **180**, 335 (1986); *ibidem*, *Nuclear Physics B* **310**, 743 (1988).
- [14] J. A. S. Lima and J. M. F. Maia, *Phys. Rev D* **49**, 5597 (1994); J. A. S. Lima and M. Trodden, *Phys. Rev. D* **53**, 4280 (1996), [[astro-ph/9508049](#)]; J. M. F. Maia and J. A. S. Lima, *Phys. Rev D* **65**, 083513 (2002), [astro-ph/0112091](#).
- [15] M. Turner, *Phys. Rev. D* **28**, 1243 (1983).
- [16] E. W. Kolb and M. Turner, “The Early Universe”, Addison Wesley (1990).
- [17] K. Olive, G. Steigman and T. P. Walker, *Phys. Rept.* **333**, 389 (2000). See also G. Steigman, [arXiv:1008.4765 \[astro-ph.CO\]](#) (2011).
- [18] G. Gibbons, & S. W. Hawking, *Phys. Rev. D.* **15**, 2738, (1977).
- [19] M. Kowalski *et al.*, *Astrophys. J.* **686**, 749 (2008), [[arXiv:0804.4142](#)]. (Union data).
- [20] P. J. E. Peebles, “Toward a new cosmological paradigm”, in *AIP Conf. Proc.* 1241, 175, (2010), [arXiv:0910.5142 \[astro-ph.CO\]](#).
- [21] E. Komatsu, et al., *Astrophys. J. Suplem.* **192**, 18 (2011).
- [22] J. A. S. Lima, J. F. Jesus and F. A. Oliveira, *JCAP* **11**, 027 (2010), [[arXiv:0911.5727](#)].
- [23] S. Basilakos & J. A. S. Lima, *Phys. Rev. D* **82**, 023504 (2010), [[arXiv:1003.5754](#)]. See also, Jesus et al. (2001), [arXiv:1105.1027 \[astro-ph.CO\]](#).
- [24] E. Gunzig, J. G  h  niau and I. Prigogine, *Nature* **330**, 621 (1987).
- [25] S. Weinberg, *Rev. Mod. Phys.*, **61**, 1 (1989).